How do mathematicians write and evaluate conditional statements?

Hypothesis
Given
Conclusion
Implied

If $\underbrace{x=5}_{H}$, then $\underbrace{x^2=25}_{C}$.

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$\underbrace{x^2=25}_{C}$ when $\underbrace{x=5}_{H}$.

$\underbrace{x=5}_{H}$ implies $\underbrace{x^2=25}_{C}$.

☆ $x=5 \longrightarrow x^2=25$

Converses

The converse of a conditional statement exchanges the hypothesis and the conclusion.

Example 1: Original Statement (OS)

If $\underbrace{x=5}_{H}$, then $\underbrace{x^2=25}_{C}$.

Is this statement true?

$x=5$ $\quad (5)^2=25$
$25=25$ $\boxed{\text{True!}}$

Converse Statement (CS)

If $\underbrace{x^2=25}_{H}$, then $\underbrace{x=5}_{C}$.

Is this statement true?

$x^2=25$
$x=\pm 5$

$\boxed{\text{False}}$

Counter Example (CE)

$x=-5$
$\underset{T}{(-5)^2=25}$ but $\underset{F}{-5=5}$.

☆ Counterexample: The hypothesis is true, but the conclusion is false!

Ex 2: OS - If $\underbrace{M \text{ is the midpoint of } \overline{AB}}_{H}$, then $\underbrace{AM = MB}_{C}$.

Is this statement true?

$\boxed{\text{True}}$ [midpt. Thrm]

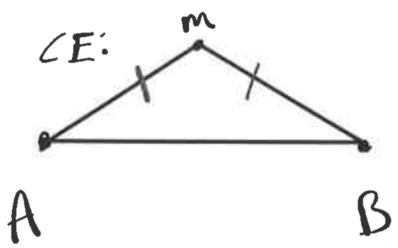
What is the converse of this statement?

CS - If $\underbrace{AM = MB}_{H}$, then $\underbrace{M \text{ is the midpt of } \overline{AB}}_{C}$.

Is this statement true?

$\boxed{\text{False}}$

CE:



$\underbrace{AM = MB}_{T}$ but M is <u>not</u> the midpt of $\overline{AB}$.

Ex 3: OS - If $\underbrace{2x-1=5}_{H}$, then $\underbrace{x=3}_{C}$.

Is this statement true?

$2x-1=5$
$2x=6$
$x=3$

$\boxed{\text{True}}$

What is the converse of this statement?

CS - If $\underbrace{x=3}_{H}$, then $\underbrace{2x-1=5}_{C}$.

Is this statement true?

$x=3$ $\quad 2(3)-1=5$
$6-1=5$

$\boxed{\text{True}}$

☆ If a conditional statement and its converse are both true, you can write a biconditional statement.

$2x-1=5$ if and only if $x=3$.

iff

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